Critical exponents of the $O(N)$ model in the infrared limit from functional renormalization

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We determined the critical exponent ν of the scalar $O(N)$ model with a strategy based on the definition of the correlation length in the infrared limit. The functional renormalization group treatment of the model shows that there is an infrared fixed point in the broken phase. The appearing degeneracy induces a dynamical length scale there, which can be considered as the correlation length. It is shown that the IR scaling behavior can account either for the Ising type phase transition in the 3-dimensional $O(N)$ model, or for the Kosterlitz-Thouless type scaling of the 2-dimensional $O(2)$ model.

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I. INTRODUCTION

The renormalization group (RG) method is a powerful tool to take into account all the quantum fluctuations systematically [1, 2]. The RG flow equations constitute a bridge between the high energy, microscopic, ultra-violet (UV) physics and the low-energy, infrared (IR) one. One usually starts with a UV potential, which describes the small distance interactions among the elementary excitations and, by taking into account the quantum fluctuations in a consecutive manner, one finally arrives at the effective theory which contains the complete effects of the quantum modes.

The RG flow equations are highly non-linear and, due to the complexity of initial conditions, it is difficult to map out the whole phase space, therefore one should use some approximations. In the vicinity of the fixed points of the RG equations one can linearize the flow equations which can make the physical problem analytically treatable there. The 3-dimensional (3d) $O(N)$ symmetric scalar model contains a Gaussian and a Wilson-Fisher (WF) fixed point. Novel approaches and new improvements of the RG method are usually tested by calculating the scaling exponents in the vicinity of its non-trivial WF fixed point. The exponents can be calculated numerically by field expansion of the potential [3–5], or e.g. by ϵ-expansion in $4-\epsilon$ dimensions [6], furthermore one can investigate the convergence the value of the exponents in the derivative expansion [7]. In the framework of the RG method more precise results can be obtained without expanding the potential [8, 9]. The exponents can also be calculated by improved Hamiltonian for the 3d Ising model [10].

Our goal is to show that there is an IR fixed point in the d-dimensional $O(N)$ models inducing an IR scaling regime, which enables one to determine the correlation length and the critical exponents there. Since the IR fixed point is attractive, the scaling of the correlation length ξ cannot be determined with the usual technique according to the linearization around the saddle-point-type WF fixed point. We use an non-conventional strategy in order to define ξ which is based on the IR behavior of the RG flows in the broken phase [11, 12]. It was shown [13–15] that at a certain momentum scale the RG evolution stops that defines a maximal length scale which can characterize the appearing global condensate there. This scale can be identified by the correlation length ξ. It diverges as a power law-function which can provide us the critical exponent ν of ξ in the IR limit. This method enabled one to determine the value of ν of the 3d scalar $O(1)$ model, and to recover the scaling of the Kosterlitz-Thouless (KT) -type phase transition in the sine-Gordon (SG) model, too [11].

We use the functional RG treatment for the d-dimensional $O(N)$ model for its effective average action in order to get the deep IR scaling behavior of the effective potential. We show that the exponent ν based on the power law scaling of the correlation length in the IR limit for the 3d $O(N)$ model and for the $O(1)$ model with continuous dimension coincides with the one, which can be obtained around the WF fixed point by the conventional method based on finding negative reciprocal of the eigenvalue corresponding to the single relevant operator. We also show that the other critical exponents are not necessarily equal in these regions, e.g. the exponent η calculated in the IR scaling regime is much larger than its value determined in the vicinity of the WF fixed point. Furthermore, we investigated the 2d $O(2)$ model, where instead of the WF fixed point a critical slowing down of the evolution appears. We obtained that the IR scaling reproduces properly the KT-type scaling of the model in a very simple way. It is important to note that our treatment can determine the value of the exponent ν for the KT-type scaling directly, which is in principle not possible in the conventional way.

The paper is organized as follows. In Sect. II we give the RG evolution equation of the scalar $O(N)$ model. In Sect. III the typical phase structure of the $O(N)$ model is presented and the appearing fixed points are discussed. The results for the exponent ν for the 3d $O(N)$ model and for the $O(1)$ model with continuous dimension are shown in Sects. IV and V. We recover the essential scaling of the correlation length in the IR limit for the 2d $O(2)$ model

in Sect. VI, and finally, in Sect. VII the conclusions are drawn up.

II. RENORMALIZATION

The successive elimination of the quantum fluctuations is performed by means of the Wetterich RG equation for the effective action [1]

$$k\partial_k\Gamma_k = \frac{1}{2}\mathrm{Tr}\frac{k\partial_k R_k}{R_k+\Gamma_k''} \tag{1}$$

where the prime denotes the differentiation with respect to the field variable ϕ, and the trace Tr denotes the integration over all momenta and the summation over the internal indices. Eq. (1) has been solved over the functional subspace defined by the ansatz

$$\Gamma_k = \int_x \left[\frac{Z}{2}\partial_\mu\phi^a\partial^\mu\phi_a + V\right], \tag{2}$$

with the wavefunction renormalization Z and the potential V, which are functions of the invariant $\rho = \phi^a\phi_a/2$. The potential has the form

$$V = \sum_{n=2}^{M}\frac{g_n}{n!}(\rho - g_1)^n, \tag{3}$$

with M the degree of the Taylor expansion and the dimensionful couplings g_n, $n \geq 1$. One can introduce the dimensionless couplings according to $g_1 = k^{[g_1]}\kappa$ and $g_n = k^{[g_n]}\lambda_n$, with $[g_1] = d-2$ and $[g_n] = d+n(1-d/2)$ for $n \geq 2$. For shorthand we use $\lambda_2 = \lambda$. The further dimensionless quantities are denoted by $\sim$, e.g. $V = k^d\tilde{V}$, thus

$$\tilde{V} = \sum_{n=2}^{M}\frac{\lambda_n}{n!}(\rho-\kappa)^n. \tag{4}$$

The introduction of the coupling κ serves a better convergence in the broken phase. The IR regularization is chosen to be the Litim's regulator [16, 17]

$$R_k = (k^2-q^2)\theta(k^2-q^2), \tag{5}$$

which gives fast convergence during the evolution, and provides simple analytic forms for the flow equations. The evolution equation for the potential can be derived from Eq. (1) [3], which reads as [16]

$$\begin{aligned} k\partial_k\tilde{V} &= -d\tilde{V} + (d-2+\eta)\tilde{\rho}\tilde{V}' + \frac{4v_d}{d}\left(1-\frac{\eta}{d+2}\right) \\ &\times\left(\frac{1}{1+\tilde{V}'+2\tilde{\rho}\tilde{V}''} + \frac{N-1}{1+\tilde{V}'}\right), \end{aligned} \tag{6}$$

with the Litim's regulator in Eq. (5), where $' = \delta/\delta\rho$ and $v_d = 1/\Gamma(d/2)2^{d+1}\pi^{d/2}$, with the Gamma function $\Gamma(d/2)$. In Eq. (6) we introduced the anomalous dimension η which is defined as $\eta = -d\log Z/d\log k$ and can be calculated by means of the couplings as

$$\eta = \frac{16v_d}{d}\frac{\kappa\lambda^2}{1+2\kappa\lambda} \tag{7}$$

if we consider the Litim's regulator. The inclusion of η in the RG equation accounts for the evolution of the wavefunction renormalization. We note that a more precise treatment can be obtained if one Taylor expand Z as the potential V in Eq. (4), and considers its evolution equation which has a similar but more involved structure as the potential has in Eq. (6). We also note that the inclusion of the full momentum dependence of the wavefunction renormalization [18] would make the behavior of the anomalous dimension regular, which could show that the obtained scaling of η is the consequence of the approximations used here.

From the functional RG equation in Eq. (6) one can deduce evolution equations for the couplings κ, λ and λ_n, $n \geq 2$, e.g. in $d = 3$ the flow equations are

$$\begin{aligned} k\partial_k\kappa &= -\kappa + \frac{1}{2\pi^2(1+2\kappa\lambda)^2} \\ k\partial_k\lambda &= -\lambda + \frac{3\lambda^2}{\pi^2(1+2\kappa\lambda)^3} \end{aligned} \tag{8}$$

for the first two couplings if we set $\eta = 0$ and $\lambda_n = 0$ for $n > 2$. The scale k covers the momentum interval from the UV cutoff Λ to zero. In numerical calculations we typically set $\Lambda = 1$. The dependence of the number of couplings should always be considered [3, 4, 19]. We also investigated the M dependence of our numerical results, and we obtained that the choice $M = 8$ gives stable results for the exponents in the IR limit if we Taylor expand the potential.

III. THE PHASE SPACE, FIXED POINTS

The $O(N)$ model in $d = 3$ has two phases. The typical phase structure is depicted in Fig. 1 for the couplings in Eq. (8). There are two fixed points in the model, which can be easily identified from the RG equations in Eq. (8) if one solves the static equations. The Gaussian fixed point is situated near the origin at $\kappa_G^* = 1/2\pi^2$ and $\lambda_G^* = 0$. The linearization of the flow equations around the Gaussian fixed point gives a matrix with positive eigenvalues ($s_{G1} = 1$ and $s_{G2} = 1$) for the couplings showing that this fixed point is repulsive, or UV attractive. The WF fixed point can be found at the values of the couplings $\kappa_{WF}^* = 2/9\pi^2$ and $\lambda_{WF}^* = 9\pi^2/8$, with eigenvalues $s_{WF1} = 1/3$ and $s_{WF2} = -2$. The opposite signs of the eigenvalues refer to the hyperbolic, or saddle point nature of the fixed point with attractive and repulsive directions in the phase space.

We usually identify the critical exponent ν of the correlation length ξ in the vicinity of the WF fixed point

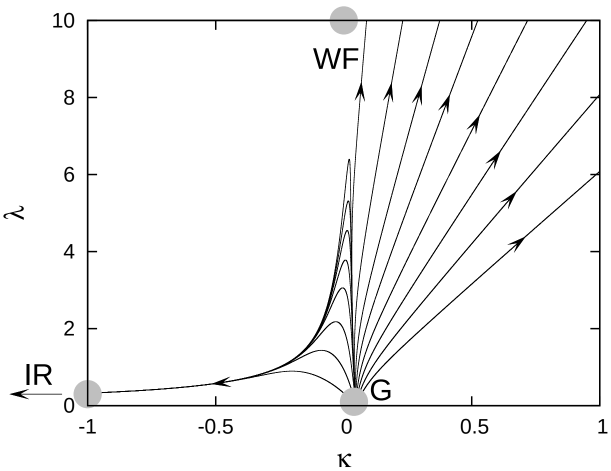

FIG. 1: The phase space of the 3d $O(N)$ model. The flows belonging to the symmetric (broken) phase tend to right (left), respectively.

by taking the negative reciprocal of the single negative eigenvalue, which gives $\eta_{WF} = 1/2$ in this case. For many couplings we should find the fixed points by solving the static RG equations numerically. These equations are non-linear, and it is very difficult to find them in an M dimensional phase space, especially if one does not know in which region of the space space they can be found. Conventionally the fixed points are found by e.g. shooting method where the non-expanded potential is considered. However this technique is inappropriate for models where only a critical slowing down appears in the RG flow, since no fixed points can be found. This situation takes place e.g. in the 2d $O(2)$ model.

It is worthwhile to mention that the flows tend to a single curve beyond the WF fixed point. In the broken phase one obtains a (super)universal effective potential. The phase structure suggests in Fig. 1 that a further fixed point may exist in the IR limit. The IR effective potential in the broken phase can be characterized by an upside down parabola [13, 14] which implies that $\kappa^*_{IR} \to -\infty$ and $\lambda^*_{IR} = 0$, therefore the IR limit should be found there. Mathematically it seems that there are no further fixed points of the RG equation in Eq. (8), but the values for κ^*_{IR} and λ^*_{IR} could satisfy the static equations, although they give expressions like $\infty - \infty$ and $0/0$. However if one rescales the couplings as in [20, 21] then the attractive IR fixed point can be uncovered. If one repeats those calculation for the couplings in Eq. (8) one finds the following pair of evolution equations

$$\begin{aligned} \partial_\tau \omega &= 2\omega(1-\omega) - \frac{\ell\omega}{\pi^2}(3-4\omega) \\ \partial_\tau \ell &= \ell(5\omega - 6) + \frac{9\ell^2}{\pi^2}(1-\omega), \end{aligned} \tag{9}$$

where $\omega = 1 + 2\kappa\lambda$, $\ell = \lambda/\omega^3$ and $\partial_\tau = \omega k \partial_k$. The static equations now have the Gaussian ($\ell^*_G = 0$, $\omega^*_G = 1$), the WF ($\ell^*_{WF} = \pi^2/3$, $\omega^*_{WF} = 3/2$) and the IR ($\ell^*_{IR} = 2\pi^2/3$, $\omega^*_{IR} = 0$) fixed point solutions. Naturally the Gaussian and the WF ones has the same behavior as was obtained from direct calculations. However the new IR fixed point indeed corresponds to the values $\kappa^*_{IR} \to -\infty$ and $\lambda^*_{IR} = 0$, and the linearization in its vicinity gives a negative and a zero eigenvalue, showing that the fixed point is IR attractive, in accordance with the flows in Fig. 1.

If one considers more couplings and includes the running of the anomalous dimension one obtains qualitatively similar results with similar phase space structure as in Fig. 1 and with the same fixed points. However, in the broken phase the effective potential has a wide range of flat region in its middle, implying that the Taylor expansion of the potential as in Eq. (4) does not converge well. One can obtain more reliable results if one considers the evolution of the non-expanded potential in Eq. (6) without a polynomial ansatz. Nevertheless we continue our investigation by expanding the potential, similarly to [4, 5, 16], since our goal is to demonstrate the existence of the IR fixed point and to calculate the corresponding exponents in its vicinity, and not to get better exponents than the ones obtained by high precision Monte-Carlo calculations [22, 23].

The RG flow for M number of couplings can also drive the evolution in Eqs. (6) and (7) to degeneracy where $\omega = 0$. The r.h.s. of the RG flow equations become singular, when the degeneracy condition $\omega = 0$ is satisfied at a certain value of $k = k_c$. As k approaches k_c the anomalous dimension η becomes also singular and it tends to zero at k_c abruptly, as it is shown in the upper figure in Fig. 2, which also demonstrates that there are three different scaling regions in its flow. In order to analyze them we plotted η as the function of the shifted scale $k - k_c$ in the lower figure in Fig. 2. In the UV region the Gaussian fixed point drives the evolution of the anomalous dimension. In this regime it grows according to the power law scaling $\eta_{UV} \sim (k - k_c)^{-2}$. There is a crossover (CO) scaling region between $10^{-8} \lesssim k - k_c \lesssim 10^{-4}$ where a plateau appears giving a constant value for $\eta_{CO} \approx 0.043$ due to the WF fixed point. Going further in the evolution towards the smaller values of k below $k - k_c \sim 10^{-8}$ one can find a third scaling regime. It appears a simple singularity in the upper figure, but the shifted scale $k - k_c$ clearly uncovers the power law scaling of the anomalous dimension there according to $\eta_{IR} \sim (k - k_c)^1$. This scaling region is induced by the IR fixed point.

The evolution of the other couplings also shows such type of scaling regimes with similar singularity structure in the IR limit. There the power law scaling behaviors also takes place as the function of $k - k_c$ with the corresponding exponents. This shows that the appearing singularities are not artifacts and the RG flows can be traced down to the value of k_c.

We note that one can find such a value of M where the evolution does not stop as in the upper figure in Fig. 2, but after the sharp fall η continues its RG evolution marginally giving a tiny value of η there. However, the singular-like fall possesses the same power-law

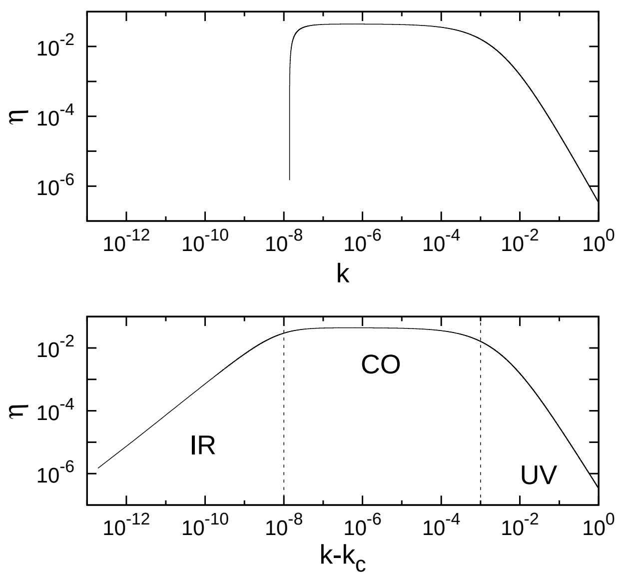

FIG. 2: The evolution of the anomalous dimension η is presented as the function of the scale k (top) and the shifted scale $k-k_c$ (bottom). The flow of η has a strong singularity as the function of the scale k at $k=k_c$, but it shows a power law like behavior as the function of the shifted scale $k-k_c$.

like behavior as the function of $k-k_c$, for any value of M. It suggests that the value of η rapidly falls to zero at k_c and it is due to the numerical inaccuracy, whether the RG evolution survives the falling and can be traced to any value of k, or the flows stop due to the appearing singularity. It strongly suggests that the singular behavior with its uncovered IR scaling for the shifted scale $k-k_c$ is not an artifact but is of physical importance. We note that one has the same singularity if one solves the RG equation in Eq. (1) without any functional ansatz for the potential [9]. The RG flows stop at k_c due to the huge amount of soft modes close to the degeneracy. It implies that during the evolution a dynamical momentum scale is generated, which can be identified by the characteristic scale of the global condensate in the broken phase. When the UV value of κ_Λ is fine tuned to its critical value κ^*_Λ the dynamical scale k_c tends to zero. The reciprocal of k_c can be identified by the correlation length ξ of the model. Such an identification of ξ ensures that the analysis is performed in the vicinity of the IR fixed point. In the IR limit ξ diverges similarly to what it does in the presence of any other fixed point, so the IR physics of the broken phase possesses an IR fixed point.

In Fig. 3 the quantity $\omega = 1+2\kappa\lambda$ is plotted for various initial conditions. It shows that in the broken phase, where $\omega \to 0$ the flows break down determining the scale $\xi = 1/k_c$ as the size of the appearing condensate. In the symmetric phase ω remains finite during the evolution and gives diverging flows in Fig. 3 according to power-law behavior in the IR. One can identify the reduced

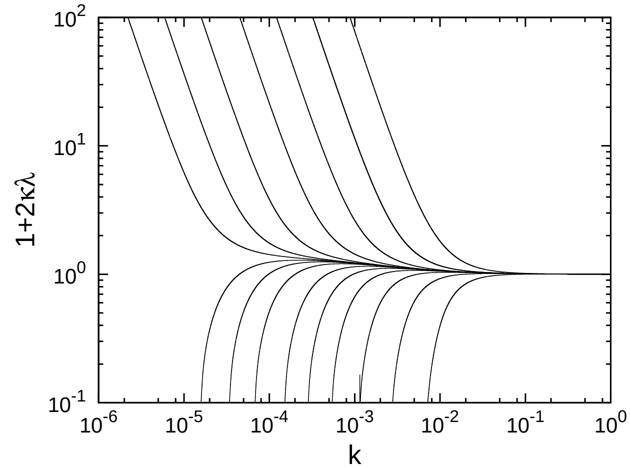

FIG. 3: The scaling of the degeneracy condition $1+2\kappa\lambda$ as the function of the scale k. The diverging curves correspond to the symmetric phase, while the curves tending sharply to zero correspond to the broken phase.

N	1	2	3	4	10	100
$\nu_{\rm IR}$	0.624	0.666	0.715	0.760	0.883	0.990
$\nu_{\rm WF}$	0.631	0.666	0.704	0.739	0.881	0.990

TABLE I: The critical exponent ν in the $O(N)$ model for various values of N.

temperature t in the $O(N)$ model as the deviation of the UV coupling κ_Λ to its UV critical value, i.e. $t \sim \kappa^*_\Lambda - \kappa_\Lambda$. The correlation length has the power law scaling behavior according to

$$\xi \sim t^{-\nu}, \tag{10}$$

which characterizes a second order phase transition. The IR defined ξ gives us a possibility to recalculate the exponent ν in the vicinity of the IR fixed point.

IV. 3d $O(N)$ MODEL

High accuracy calculations exist [22, 23] for the calculation of the exponent ν in the vicinity of the WF fixed point for the 3d $O(N)$ model. Our goal is now to get ν from the scaling around the IR fixed point. We numerically calculated the scaling of the couplings and then we determined the critical scale $k_c = 1/\xi$ for given UV values of κ_Λ. The results are plotted in Fig. 4. For a given value of N we obtain power-law behavior for the scaling of ξ, and the slope of the lines provides the exponent ν in the log-log scale. The obtained results are listed in Table I. We denoted the WF (IR) values of ν as $\nu_{\rm WF}$ ($\nu_{\rm IR}$), respectively. The results show high coincidence. The values $\nu_{\rm WF}$ are taken from results obtained from derivative expansion up to the second order, since this approximation are the closest to our treatment. One

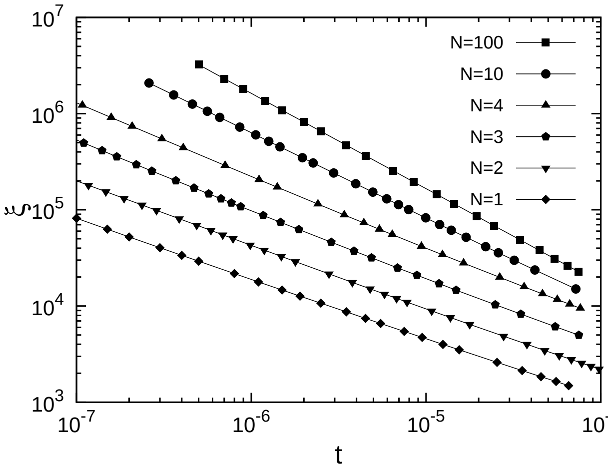

FIG. 4: The scaling of the correlation length ξ as the function of the reduced temperature t, for various values of N.

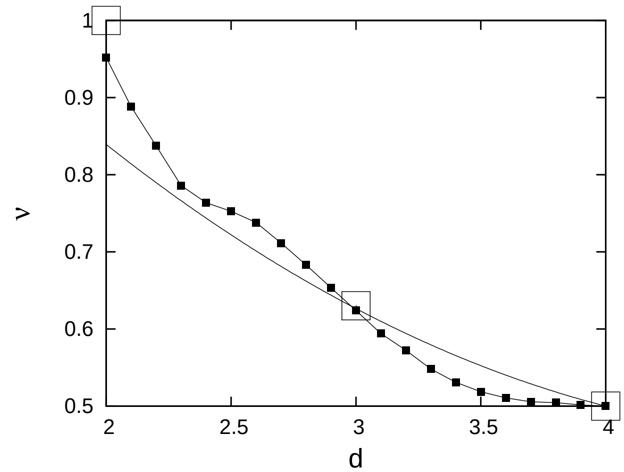

FIG. 5: The value of the critical exponent ν for various dimension d (filled squares). The continuous curve obtained from ϵ-expansion [24]. The empty squares denote the exact values at integer dimensions.

can conclude that the exponent ν can be also determined from the scaling around the IR fixed point, and has the same value as was obtained around the WF fixed point. This fact might seem quite strange at a first glance since the two fixed points are well separated in the phase space according to Fig. 1. The coincidence may come from the fact that the condensate of the broken phase has a global feature which accompanies the whole flow starting from UV to IR. The other exponents do not necessarily coincide, e.g. anomalous dimension $\eta \approx 0.04$ in the vicinity of the WF fixed point, while it converges to zero in the IR one.

V. DIMENSION-DEPENDENCE OF ν IN THE $O(1)$ MODEL

As a further test we calculated the exponent ν for the $O(1)$ model for continuous dimension. By using ϵ-expansion the dimension dependence of ν can be easily obtained [24, 25]. By linearizing the functional RG flow equations around the WF fixed point the exponent ν can be calculated for any dimensions [26]. Naturally far from $d = 4$ the anomalous dimension grows up, which requires to take into account the evolution of the wavefunction renormalization.

We determined the exponent ν by the degeneracy induced scaling around the IR fixed point. The results are shown in Fig. 5. The exponent ν calculated around either the WF or the IR fixed points give the same results. The square at $d = 2$ in Fig. 5 shows $\nu = 1$, as the exact results of the 2d Ising model [27]. When $d = 3$ there are no exact calculations but one can find a huge amount of articles related to calculating ν [22, 23]. When $d = 4$ the mean field calculations give $\nu = 0.5$. Our results give better results than the ϵ-expansion ones for low dimensions, since far from $d = 4$ one needs more and more loop corrections to get reliable results. At around $d \approx 2.5$ there is a wiggle in Fig. 5, which does not appear in [26]. This is due to the numerical inaccuracy in our method coming from the high anomalous dimension in the IR regime which should be traced to extremely small values of the scale k. We expect that by taking into account higher orders in the derivative expansion or by treating the problem with full momentum dependence [18] the wiggles could be removed.

These results clearly show that it is possible to determine the power-law scaling of the correlation length in the vicinity of the IR fixed point. We note that there is no WF fixed point in case of $d = 4$ model. However the degeneracy induced scaling gives us back the analytic result even in this case, too.

VI. 2D $O(2)$ MODEL

The 2-dimensional $O(2)$ model is exceptional among the $O(N)$ models since it possesses an infinite order KT-type phase transition [28]. This transition is characterized by the scaling of the correlation length according to

$$\log \xi \sim t^{-\nu}, \tag{11}$$

with the exponent $\nu = 0.5$. The 2d $O(2)$ model belongs to the same universality class as the 2d XY model [27], the 2d Coulomb gas [29] the 2d SG model [30], but the equivalent models inherently contain vortices as elementary excitations which can account for the essential scaling. The functional RG approach shows KT-type scaling according to Eq. (11) if one considers the effect of the wavefunction renormalization in the SG model [21], furthermore it was found that the KT and the IR scaling gives the same scaling behavior and the same exponent $\nu \approx 0.5$ [11]. However the 2d $O(2)$ model does not contain

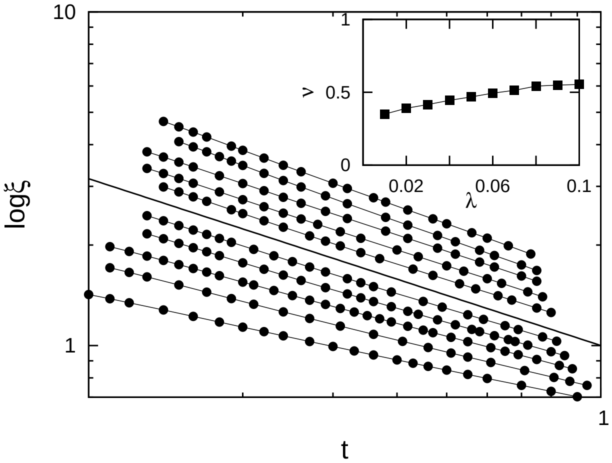

FIG. 6: The KT-type scaling of the correlation length in the $O(2)$ model in $d = 2$. The lines correspond to flows belonging to different UV values of λ.

any vortices but the scaling in Eq. (11) can be uncovered [31, 32], although in a rather involved way.

The 2d $O(2)$ model also contains an IR fixed point, therefore the degeneracy induced IR scaling also can be used. We plotted the results in Fig. 6. They show that we can recover the essential scaling according to Eq. (11) in a very simple way by using our proposed technique, although the exponent shows a slight dependence on the UV value of λ as is depicted in the inset of Fig. 6. Besides, one can conclude that $\nu \approx 0.5$, in accordance with other results [21, 31, 33].

VII. SUMMARY

The critical exponent ν of the correlation length ξ was calculated for the $O(N)$ model in the IR limit. We showed that there is an IR fixed point in the broken phase of the model, which induces a degeneracy and stops the RG evolution at a finite momentum scale. The latter defines a characteristic length scale which can be identified with the correlation length. This technique of identifying the correlation length enabled us to determine the exponent ν around the IR fixed point, far from the WF one in which vicinity it is usually calculated. We showed that the value of the exponent ν agrees well with results obtained in the vicinity of the WF fixed point for the 3d $O(N)$ model, see Table I. The qualitative behavior of ν for the $O(1)$ model with continuous dimension presented earlier by ϵ-expansion or linearization around the WF fixed point was recovered by the IR scaling. Furthermore the essential scaling of the 2d $O(2)$ model with the exponent $\nu \approx 1/2$ was also got back. The latter model exhibits a KT-type phase transition showing that this simple technique can be easily applied to describe any types of phase transitions.

The coincidence of the exponent ν around the WF and the IR fixed point may come from the fact that the correlation length in the broken phase characterizes the condensate which occurs there and consists of a macroscopic population of soft modes showing the global nature of ξ. The anomalous dimension η does not agree around the WF and the IR fixed points. The common feature of the d-dimensional $O(N)$ models for $d > 2$ that there is a crossover fixed point, namely the WF fixed point there. Although the crossover and the IR fixed points are far from each other in the phase space, we read off the scaling of ξ from the same trajectories which first approach the crossover fixed point and then tend towards the IR one. However the real power of this technique can emerge if there are no crossover scalings. For example in the 2d $O(2)$ model there is only some remainder of the WF fixed point as a critical slowing down of the RG flows, or in the 4d $O(1)$ model the WF fixed point melts into to the Gaussian one. It is shown that the IR scaling can correctly determine the order of the phase transitions, and the proper value of the exponents ν in these models, too. A further non-trivial example is the massive SG model [34], and the layered SG model [11, 35] where there are no crossover fixed points too, but the IR scaling uncovers the type of the phase transition and helps us to determine in which universality class the models belong to [36].

Acknowledgments

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